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BALANCING FOR IDENTIFICATION AND CONTROL

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The purpose of this note is to motivate the need for a new concept of balancing which is particularly suitable for identification based control.

Balanced realizations have very interesting properties and are particularly useful for model reduction. However the direct identification of reduced order models that have been obtained from a balanced realization is still an open problem. Progress to couple balancing and identification for the full order modell has been made possible by the Ober-parametrization and its extensions (cf. [10, 9, 6, 8]). The main feature of this parametrization from the identification point of view is that the *parameter space* is the positive orthant (except for a number of structural parameters which identify a local chart), and thus the truncation or resetting mechanism that is to be used in a recursive estimation scheme is very simple. It should be noted that using continuous-time identification methods, the Ober-parametrization is directly identifiable, and the bilinear transformation to couple discrete and continuous time models can be avoided. Balanced realizations have interesting ***sensitivity minimization*** properties (cf [7]), but these seem to be counterproductive from the identification point of view. In this paper we present a criterion against which the goodness of the Ober-parametrization can be evaluated and the need for a new concept of balanced parametrization is introduced. This criterion is obtained by considerations related to the theory of stochastic complexity. Similar ideas in another context were given in [8], where the need for ***scaling and preconditioning*** of the data are mentioned. An important technical component of the theory of ***stochastic complexity*** is the analysis of the effect of ***parameter uncertainty*** on prediction (cf. [11, 4, 2]). We shall quote a key result of [4] in a sightly more general form as follows: let (y_n) be a second order stationary process , given by the state space system:

$$x_{n+1} = A(\theta^*)x_n + B(\theta^*)e_n$$

$$y_n = C(\theta^*)x_n + e_n \qquad (LSS)$$

which is assumed to be stable and inverse stable. Thus (e_n) is the innovation process of (y_n), $\sigma^2 = \sigma^2(e)$. Assume that (e_n) is L-mixing (cf. [4]), and assume that the parametrization of the system matrices is smooth and identifiable, and let $k = \dim\ \theta^*$. For any assumed value θ of θ^* in a feasible domain we can invert the above system and get an estimated innovation sequence $\varepsilon_n(\theta)$. Let the prediction error estimator of θ^* based on $y_1, \ldots, y_n$ be $\widehat{\theta}_n$. For a rigorous definition of it cf. [3]. Then we have the following result:

Theorem 1 *We have*

$$\mathrm{E}(\varepsilon_n^2(\widehat{\theta}_{n-1}) - e_n^2) = \sigma^2 \frac{k}{n}(1 + o(1)).$$

The proof of the theorem is more relevant for the present paper than the statement itself. The *main idea* is to approximate the left hand side by

$$\mathrm{E}(\widehat{\theta}_{n-1} - \theta^*)^T \varepsilon_{\theta n}(\theta^*) \varepsilon_{\theta n}^T(\theta^*)(\widehat{\theta}_{n-1} - \theta^*) \simeq \frac{1}{2}\mathrm{Tr} T^*(R^*)^{-1},$$

where

$$T^* = \mathrm{E}\varepsilon_{\theta n}(\theta^*)\varepsilon_{\theta n}^T(\theta^*), \qquad R^* = \mathrm{E}(\widehat{\theta}_{n-1} - \theta^*)(\widehat{\theta}_{n-1} - \theta^*)^T.$$

Here the weak dependence of $\varepsilon_{\theta n}(\theta^*)$ and $(\widehat{\theta}_{n-1} - \theta^*)$ is exploited. The matrix T^* is a second order sensitivity matrix, while $(R^*)^{-1}$ is the covariance matrix of the estimator. Obviously the accuracy or quality of the prediction will depend on the *condition numbers* and relative magnitude of T^* and R^*, which in turn depend on the parametrization of the system.

A similar result has recently been obtained for certain linear stochastic control systems. Let us consider

$$A^* y = q^{-1} b^* u + e$$

where $\deg A^* = p$. The *minimum variance controller* of this system is obtained by

$$u_n = -\sum_{i=1}^{p} k_i^* y_{n-i+1}, \quad k_i^* = -a_i^*/b_i^*.$$

With this controller we get $y_n = e_n$. The Åström-Wittenmark regulator (c.f. [1] or [12]) generates recursive estimators of k_i^* which is then applied in the control loop. Thus we get an output process (y_n) which is slightly different from (e_n).

Theorem 2 *Under suitable technical conditions we have*

$$\mathrm{E}(y_n^2 - e_n^2) = \sigma^2(e)\frac{p}{n}(1 + o(1)).$$

Again the left hand side can be approximated by an expression $\sigma^2 \mathrm{Tr} T^*(R^*)^{-1}$, where $T^* = \mathrm{E}\dfrac{\partial^2}{\partial\theta^2} y^2(\theta)$. Here $y(\theta)$ is the output that we get by the minimum variance controller, assuming θ to be the true parameter, and R^* is the estimation covariance matrix. Similar theorems can be proved for nonstandard parametrizations, but then we get different T^* and R^*.

The situation is more dramatic for continuous time systems. The effect of parameter uncertainty onto prediction say τ-time ahead in the context of continuous time models and continuous time identification can be given by a trace formula $\frac{1}{n}\mathrm{Tr} T^*(R^*)^{-1}$ just as in Theorem 1 (cf. [5]). However, if we solve the problem of prediction by fitting a *discrete-time model* to our data and then performing a (τ/h)-step-ahead prediction using estimated discrete-time parameters, then $(R^*)^{-1}$ and T^* will be replaced by $h^2(R^*)^{-1}$ and $h^{-2}T^*$, which results in *extreme numerical sensitivity* when computing the predictor. We conclude that continuous time modelling and parametrization is to be preferred for high accuracy prediction of continuous-time processes. Going back to our first example the above considerations lead us to the following:

Definition A parametrization of (LSS) is called *sensitivity-identifiability balanced* if $R^* = T^* = \sigma I$.

The existence of such parametrizations is an open question. From the practical point of view approximate balancing would be satisfactory.

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